Long Barotropic Waves Generated by a Storm Crossing Topography

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ABSTRACT

Storms crossing topography are shown to radiate long surface gravity waves. The waves are transients generated by changes in the depth-dependent amplitude of the atmospherically forced pressure wave beneath a storm. This generation mechanism for long waves, known as “meteorological tsunamis” or rissaga, does not appear to have been previously discussed. The transients have periods equal to the passage time of the storm, of order 30 min for small fast-moving storms. A 1D model is used to give the amplitudes of the transient waves generated by a small fast-moving storm crossing a topographic step on to a continental shelf and across a ridge. Large transients are generated by storms whose translation speed is subcritical in deep water and supercritical in shallow water, that is, faster than the shallow-water wave speed. Surprisingly, when the depth difference between the deep water and the continental shelf is large, a gentle transition from deep to shallow water over 10 storm widths only slightly reduces the amplitudes of the transients. The influence of a finite-width shelf on the enhancement of coastal storm surge is also discussed. A 2D numerical model illustrates the topographic transients generated by sub- and supercritical storms moving across a ridge. Topographic transients are suggested as a source of energy for seiches on shelves and within embayments. The energy may come from a storm crossing the adjacent continental slope and possibly from distant open-ocean storms crossing multiple ridges and seamounts.

1. Introduction

There is generally little wave energy in the surface gravity wave spectrum between long swell with periods of tens of seconds and tidal periods. The exceptions are rare energetic tsunamis generated by seismic events, which have periods of tens of minutes and seiches. Recently there have been observations of waves with periods of tens of minutes associated with fast-moving storms. For example, long waves with tsunami-like periods hit Newfoundland in 1999 and 2000. The waves resulted in flooding and could not be attributed to seismic events or local weather conditions (Mercer et al. 2002). Mercer et al. used a numerical model to demonstrate that the long-wave events may be associated with two storms crossing the Grand Banks at high speed, >30 m s\(^{-1}\). They showed that, if a storm is supercritical, that is, its translation speed is greater than the shallow-water wave speed, then it develops a barotropic wake. This wake is similar to the wave wake behind a high-speed vessel in shallow water. Mercer et al. speculated that the Newfoundland wave events were due to the wake refracted and/or reflected by the bathymetry of the Grand Banks.

The long waves associated with storms are sometimes referred to as “meteorological tsunamis” because of their having periods similar to seismic tsunamis. Around the Balearic Islands in the western Mediterranean Sea large sea level oscillations with amplitudes up to 1.5 m can occur (Monserrat et al. 1991). These waves are known locally as “rissaga” and have been attributed to forcing by low pressure systems with translation speeds near the shallow-water wave speed exciting edge waves on the islands’ continental shelf (Liu et al. 2002) and seiches within specific inlets (Rabinovich and Monserrat 1998). Large sea level oscillation events have also been observed in the Adriatic Sea (Vilibić et al. 2004), where they have been shown to be due to resonant coupling between a fast-moving storm and the seiche modes of a number of bays.

Both the Mediterranean and Newfoundland long waves are observed in harbors adjacent to continental shelves, which are near critical for fast-moving storms. Hence the waves are generated locally. This work was...
motivated by observations from the east coast of New Zealand (NZ) where 0.5-m-amplitude long wave events have been implicated in the grounding of an oil tanker (Goring 2005). The events appear to be generated by distant isolated storms in deep water 1000–2000 km east of New Zealand’s continental shelf and have wave periods of 5–30 min (Goring 2005). The NZ long waves cannot be explained by resonant coupling or as the wake of a supercritical storm as their 20 m s$^{-1}$ speed is well below the shallow-water wave speed of 200 m s$^{-1}$ for the 5000-m-deep abyssal plane northeast of NZ. The storms are moving through an area known as the Louisville Ridge, a chain of 400–1000-m-deep seamounts northeast of NZ. A secondary aim of this paper is to see if the Ridge is capable of radiating waves that travel across deep water and impact the distant New Zealand shelf.

Low air pressure within a storm elevates the ocean’s surface up to 0.5 m, creating an atmospherically forced pressure wave beneath the storm. The amplitude of this forced pressure wave is dependent on both the storm’s translation speed and the shallow-water wave speed. The amplitude of the forced wave becomes large as the storm speed approaches the shallow-water wave speed, an effect referred to as “Proudman resonance” (Proudman 1953), though it was noticed in 1879 by Lamb (1932), in article 177 on a moving disturbance. In this paper a theory for long barotropic waves generated by a storm moving over topography is developed. It is shown that long waves are transients generated by changes in the depth-dependent amplitude of the forced pressure wave beneath a moving storm. Although a seemingly classical problem, this generation mechanism does not appear to have been discussed.

The primary aims of this paper are to show that moving storms radiate long-wave transients as they cross topography and to determine some of the conditions that produce large transients. The result is to show that small fast storms radiated wave energy with periods of tens of minutes as they cross the continental slope or a ridge. It is found that except in the exceptional circumstances of near-critical storms the amplitudes of these transients are small. Though generally small their periods are similar to seiche modes of some shelves and embayments. Thus, this paper suggests topographic transients as a source of seiche energy, which may persist long after the storm and its transients have passed.

Section 2 of this paper develops a 1D analytic model for small fast storms and discusses its consequences. Section 2a gives the solution for the transient waves generated by a storm crossing a topographic step. Section 2b combines two steps together to give the transients generated by a storm crossing a ridge. Section 2c examines the limits on transient wave amplitudes, imposed by a finite-width continental shelf or ridge. Section 2d explores the implications for a coastal storm surge due to a fast storm moving across a finite-width shelf. Section 2e comments on transients generated by changes in storm intensity or its translation velocity. Section 3 gives 2D numerical solutions for idealized topography, and section 4 discusses the Newfoundland and NZ observations as well as the possibility that topographic transients are a source of seiche energy.

2. The 1D analytic model

A simple 1D barotropic model for transient shallow-water waves generated by small fast-moving storms over topography is developed. The moving storm’s time scale is $T_s = L_s/U$, the time taken for it to pass a fixed observer, where $L_s$ is the storm’s width and $U$ is its translation speed. For a fast small storm this time scale is of order an hour for $U = 30$ m s$^{-1}$ and $L_s = 100$ km. If $T_s$ is assumed to be small relative to the inertial period, then rotational effects can be neglected. For linear bottom stress the time scale of frictional decay is $h/r$, where $h$ is the water depth and $r$ is the bottom stress coefficient. In the shallowest water considered, 50 m, and for a typical $r = 0.001$ m s$^{-1}$ the frictional time scale is 13 h, much greater than the storm’s time scale; thus frictional effects may be neglected. Mercer et al. (2002) argue that weak rotational effects allow the effects of wind stress curl to be neglected and, supported by numerical tests, the effects of wind stress convergence can be neglected as long as the storm shape is nearly radially symmetric. Thus a zeroth-order model governing long waves generated by small fast storms can neglect the effects of rotation, friction, and wind stress.

In shallow water the hydrostatic balance gives the pressure at any depth $-z$ as $p = pg(\eta - z) + p_a(x, t)$, where $\eta$ is the displacement of the ocean’s surface and $p_a(x, t)$ is the atmospheric pressure at the ocean’s surface due to the storm. The atmospheric pressure can be defined in terms of the inverted barometer response as $p_a = -pg\eta_o$, where $\eta_o$ is the ocean’s surface displacement under a stationary storm. Neglecting rotational and frictional effects the linearized atmospherically forced barotropic 1D equations for shallow water motion are

\[
\frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x} + g \frac{\partial \eta_o}{\partial x} \quad \text{and} \quad \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} (h u) = 0,
\]

where $u$ is the velocity and $\eta$ is assumed to be small relative to the water depth $h$. Combining (1) and (2) for
A constant water depth gives the linear forced 1D wave equation

\[ \frac{\partial^2 \eta}{\partial t^2} - c^2 \frac{\partial^2 \eta}{\partial x^2} = -c^2 \frac{\partial^2 \eta_d}{\partial x^2}, \]  

where \( c = \sqrt{\frac{g}{h}} \). Equation (3) is a 1D version of the equation in Gill (1982) who expressed it in terms of \( \eta' = \eta - \eta_d \). If the inverted barometer response of the storm translating at constant speed \( U \) is \( \eta_d(x - Ut) \), then the steady-state forced wave solution to (3) is (Lamb 1932; Proudman 1953)

\[ \eta_d(\xi) = \frac{\eta_d(\xi)}{1 - \text{Fr}^2}, \]  

where the Froude number \( \text{Fr} = U/c \) and \( \xi = x - Ut \). The steady-state surface displacement under the storm becomes very large as \( \text{Fr} \to 1 \), that is, Proudman resonance. For \( \text{Fr} < 1 \) the forced wave is elevated above mean sea level and for \( \text{Fr} > 1 \) the forced wave is a depression. The aspect of (4) significant to this work is the dependence of the forced wave amplitude on the water depth, which appears in the denominator of \( \text{Fr} \).

As the steady-state response to the storm, \( \eta_d \), moves over changes in water depth, transient free waves are generated. These waves are generated by the interaction of the barotropic velocity associated with the forced wave with the topography. From (2) this velocity for constant water depth is given by

\[ u_s = \frac{U}{h(1 - \text{Fr}^2)} \eta_d(x - Ut). \]  

As a simple example, a storm in a constant depth ocean will generate a transient free wave as it crosses a vertical coastline. The “reflected” or reverse transient wave moves offshore and is required to satisfy the coastal boundary condition where the sum of the velocity due to the forced wave and the velocity due to the transient must be zero. The reverse transient, which satisfies this boundary condition at \( x = 0 \), is given by

\[ \eta_R = \text{Fr} \eta_d[-\text{Fr}(x + ct)] = \frac{\text{Fr}}{1 - \text{Fr}^2} \eta_d[-\text{Fr}(x + ct)]. \]  

The velocities associated with this transient are

\[ u_R = -\frac{U}{h(1 - \text{Fr}^2)} \eta_d[-\text{Fr}(x + ct)]. \]  

The \(-\text{Fr}\) factor in the argument of \( \eta_d \) in (6) and (7) is required to ensure the forced and transient barotropic velocities have the same variation in time at the coast, \( x = 0 \). The reverse transient, \( \eta_R \), has the same form as the forced wave \( \eta_d \) but is flipped about the vertical axis and stretched horizontally by a factor \( 1/\text{Fr} \), and its amplitude is scaled by \( \text{Fr} \). For \( \text{Fr} < 1 \) the reverse transient is smaller, wider, and faster than the forced wave.

This example demonstrates a number of properties of these long-wave topographic transients. Whereas a storm creates a wave wake only when \( \text{Fr} > 1 \), the transients occur for all \( \text{Fr} \). The transient amplitude relative to the forced wave increases with \( \text{Fr} \) and its amplitude relative to the inverted barometer response is large for \( \text{Fr} \approx 1 \). In this work the word amplitude is used loosely, indicating both size and sign of the forced and transient waves. The characteristic time scale for both storm and transient are the same. As the storm passes, a fixed observer sees a wave with period equal to the passage time of the storm \( T_s = L_s/\text{Fr} \). The wave period of the stretched transient moving at speed \( c \) is the same,

\[ \frac{L_s}{c} = \frac{L_s/\text{Fr}}{U/\text{Fr}}. \]  

The free waves, which constitute the wake of a supercritical storm, also have the same characteristic wave period (Mercer et al. 2002).

a. Storm crossing a step

In this section the solution for the transient waves generated by a forced wave moving from deep water onto the continental shelf across a topographic step is developed. The geometry of the step is given in Fig. 1. In the model \( x = 0 \) is at the first step and \( t = 0 \) is the time when the middle of the storm crosses the step. The steady-state responses to the moving storm (4) in deep and shallow water are given by

\[ \eta_d^s = \frac{1}{1 - \text{Fr}^2} \eta_d(x - Ut) \]  

\[ \eta_s^s = \frac{1}{1 - \text{Fr}^2} \eta_s(x - Ut), \]  

where \( S \) indicates “steady state” or storm, while \( d \) and \( s \) indicate deep and shallow water, respectively. The transition from \( \eta_d^s \) in deep water to the larger \( \eta_s^s \) on the shelf generates both a reverse transient, \( \eta_R^s \), and forward transient, \( \eta_F^s \). The subscripts indicate the reverse and forward transients generated at the first step. These waves are needed to ensure that two conditions are met at the step. First, the displacement matches across the step; that is, \( \eta_d^s + \eta_R^s = \eta_s^s + \eta_F^s \) at \( x = 0 \). Second, the mass transport matches across the step; that is, \( h_s(u_s^d + u_R^s) = h_s(u_s^s + u_F^s) \) at \( x = 0 \), where \( u \) is the horizontal velocity of the water associated with the indicated displacement from (2). The transients that sat-
Transient amplitudes are proportional to Δ, which from (8) and (9) can physically be interpreted as the fractional change in the amplitude of the steady-state forced wave as it crosses from deep to shallow water. If there is no step, Δ = 0, no transients are generated. In the limit as h_s → 0, that is, a coast, (10) equals (6).

Figure 2 gives the solution for Fr_d = 0.2 and Fr_s = 0.7 at t = 3.5T_s after a Gaussian-shaped storm crosses the step. The Gaussian storm is given by \( \eta_u = A_0 \exp[-2(x - Ut)^2/L_s^2] \), which has points of inflection L_s apart. Figure 2a gives the analytic solution, which is the sum of (8)-(11). As a check, the numerical solution to (3) was obtained (see the appendix) and it is indistinguishable from the analytic solution. The solution shows a forced wave 50% larger than it was in deep water, a very small free wave reverse transient \( \eta_{R1} \), and a much larger forward transient depression \( \eta_{F1} \). With time the forward free wave transient \( \eta_{F1} \) moves ahead of the storm to give a distinct inverted, broader copy of \( \eta_u \); \( \eta_{F1} \) moves across the shelf and is ultimately dissipated and/or reflected [(6)] at the coast.

Figure 3a plots the amplitudes of the forced and transient waves for Fr_d = 0.2 and a range of Fr_s. The small Fr_d results in a deep-water forced wave \( \eta_d^U \) only 4% larger than the inverted barometer response. The forced wave on the shelf is large for Fr_s ≈ 1 with a positive response for subcritical storms and a negative response for supercritical storms. The forward transient \( \eta_{F1} \) is also large for Fr_s ≈ 1 and the ratio of the amplitudes of the forward transient to the shallow-water forced wave amplitude \( A_{F1}/A_d^s \) given in Fig. 3a indicates the two wave amplitudes are of opposite sign. In addition, the forward transient increases in amplitude relative to the forced wave as Fr_s increases and they have the same amplitude for Fr_s = 1. Thus for this Fr_d a large forward transient is generated by a storm, which is slightly supercritical in shallow water. Figure 3a shows the reverse transient \( \eta_{R1} \) also increases with Fr_s, though its amplitude is always much smaller than the forward transient.

Large transients are generated when a slightly subcritical storm moves into slightly supercritical water depths. Figure 3b plots the amplitudes of the forced and transient waves for Fr_d = 0.8. At Fr_s = 1.2, all transient amplitudes are much larger than those for the smaller deep-water Froude number in Fig. 3a. The amplitude of the transients is proportional to Δ, for a Fr_d = 1.2 transition \( \Delta = -3.3 \) and \( A_{F1} = 3.4A_d^s \), while for Fr_d = 0.8 to Fr_d = 1.2 the transition produces larger transients as \( \Delta = -5.1 \) and \( A_{F1} = 5.5A_d^s \). The largest transient waves are generated by only a small depth change by transcritical storms whose deep- and shallow-water Froude numbers are ~1, as the forced wave must make a transition from a large elevation to a deep depression as the storm moves across the step.

In the analytical model the continental slope is assumed to be much narrower than the width of the storm and, thus, the topography can be approximated by a step. A wide continental slope might be expected to allow a gentle transition of the forced wave, signifi-
cantly reducing the amplitude of the transients. The effect on the amplitude of the transients of a wide continental slope is shown by the numerical solution in Fig. 2b. Surprisingly, the very gentle continental slope, 10\(L_s\) wide, only reduces the size of the forward transient by 20\% relative to the step solution. The reason for this is that the transient’s amplitude results from the enhancement of the forced pressure wave as it moves into shallower water. For the case shown most of this enhancement occurs on a relatively narrow region of upper slope. “Upper slope” can be quantified by defining the ratio of the forced wave’s fully developed amplitude minus its deep water amplitude to the change in amplitude between deep and shallow water:

\[
R = \left[ \frac{1}{1 - \text{Fr}_d h_s / h(x)} - \frac{1}{1 - \text{Fr}_s^2} \right] / \Delta. \tag{13}
\]

where the depth of the continental slope is \(h(x)\). Figure 4 gives \(R\) for a range of \(\text{Fr}_d\) and \(\text{Fr}_s\) for a linear continental slope. For the case in Fig. 2b, \((\text{Fr}_d, \text{Fr}_s) = (0.2, 0.7)\), 80\% of the enhancement of the forced wave occurs within the upper 20\% of the continental slope. Thus for this case, despite the very gentle continental slope in Fig. 2b, the distance spanned by the upper slope is so small that topography appears almost like a step and the reduction in transient amplitudes is small. For larger \(\text{Fr}_d\) and smaller depth changes across the

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**Fig. 2.** Ocean surface displacement for a Gaussian-shaped storm crossing a topographic step onto a continental shelf for \(\text{Fr}_d = 0.2\) and \(\text{Fr}_s = 0.7\) at \(t = 3.5T_s\) after the storm crosses the step. Dashed line gives forced wave 10\(T_s\) before reaching the shelf edge. Storm, forward, and reverse transients are labeled with their speeds. (a) Step topography (thick black line is the analytic solution; overlying gray line is the numerical solution); (b) numerical solution for wide continental slope (thick black line). Thin line is the solution for step change in depth.

**Fig. 3.** Amplitude of forced and transient waves due to a storm crossing a step relative to inverted barometer response amplitude for (a) \(\text{Fr}_d = 0.2\) and (b) \(\text{Fr}_d = 0.8\); note different scale.
slope the enhancement occurs over a wider region, reducing the size of the transients.

b. Storm crossing a ridge

A forced pressure wave moving over a ridge (Fig. 1) produces transient waves when it crosses the first step onto the ridge and when it crosses the second step back into deep water. These transients, $\eta_{R1}$, $\eta_{F1}$, $\eta_{R2}$, and $\eta_{F2}$, will be referred to as the primary transients. Figure 5a shows the forced pressure wave and the first forward transient before they hit the edge of the ridge. Figure 5b shows the waves after $\eta_{F1}$ has hit the second step and before the forced wave hits. In passing from the ridge $\eta_{F1}$ has created two new transient waves whose displacements are given by the solution to a free wave crossing a step from shallow to deep water. From Light-hill (1978) they can be expressed in terms of scaled, shifted, and stretched copies of $\eta_s$ as

$$\eta_{F1,F2} = \alpha A_{F1} \eta_s [Fr_d (x - W + c_d (t - t_2))]$$

and

$$\eta_{F1,R2} = \beta A_{F1} \eta_s [-Fr_d (x - W + c_d (t - t_2))],$$

where the transmission and reflection coefficients are

$$\alpha = \frac{2}{1 + Fr_d / Fr_s}$$

and

$$\beta = \frac{1 - Fr_d / Fr_s}{1 + Fr_d / Fr_s}$$

and $A_{F1}$ is the amplitude of $\eta_{F1}$ (Table 1); $W$ is the width of the ridge and $t_2 = W/c_s$ is the time $\eta_{F1}$ arrives at the second step. In the plotted example, the first transient radiated forward of the ridge, $\eta_{F1,F2}$, is 20% of the amplitude of the shallow-water forced wave. The reverse transient $\eta_{F1,R2}$ is of similar amplitude. When the forced wave hits the step down into deep water (Fig. 5c), two primary transients are created: the inverted $\eta_{R2}$ trails $\eta_{F1,R2}$ back across the ridge and $\eta_{F2}$, around 30% of the deep-water forced wave in amplitude, is radiated ahead of the storm. After $\eta_{F1,R2}$ hits the first step (Fig. 5d), it creates two new transients, one of which, $\eta_{F1,R2,F1}$, is radiated away from the ridge (note forward and reverse labels are given relative to the direction of the parent wave). This is the second backward transient and is much larger than the first, $\eta_{R1}$. After this, waves continue to bounce back and forth on the ridge, radiating ever smaller waves back into deep water long after the storm has crossed the ridge. Only the primary transients $\eta_{F1}$ and $\eta_{R2}$ have descendants, a few of which are given in Table 1.

Figure 6 shows the amplitude of a few of the transient waves radiated into deep water by the ridge due to the passage of a storm. Some of the transients become large if the storm is near critical over the ridge. The first two forward transients are generally of similar amplitude as indicated by their ratio $A_{F1}/A_{F2}$. The second backward transient $\eta_{F1,R2,F3}$ is of a similar amplitude to the first two forward transients and is much larger than the first backward transient $\eta_{R1}$. Again the largest transients are generated by transcritical storms.

c. Effect of shelf/ridge width

Figure 2a shows that the forced wave on the shelf develops gradually as the forward transient $\eta_{F1}$ moves ahead of the storm. To fully develop, the forced wave and transient must separate. At a given time the distance between the center of the storm and $\eta_{F1}$ due to their differing speeds is

$$\frac{\delta}{L_S} = \frac{1 - Fr_s}{Fr_s} t$$.

For the forced wave to fully develop, this distance, $\delta$, must be larger than the storm width. For example, in Fig. 2a $\delta = 1.5 L_S$ and the forced wave is nearly fully developed.

If the shelf is too narrow to allow the forced and transient waves to separate, then the elevation beneath the storm is limited. The long wave transients may also be indistinguishable from the forced wave at the coast because of the limited separation. Figure 7a shows the development of surface elevation under the storm as it moves away from the step. The development is measured by the difference between the elevation at the storm’s center and the deep-water forced wave amplitude normalized by $\Delta$. Figure 7a shows that the distance a storm needs to travel for the forced wave to become fully developed is strongly dependent on $Fr_s$. Storms
with \( F_r \) near critical require a long time, and hence distance, to fully develop the forced wave. For example, for \( F_r = 0.9 \) or 1.1 the elevation is only 50% developed six storm widths from the step. In contrast \( F_r = 0.4 \) or 1.6 requires only two storm widths to fully develop the forced wave. For near-critical storms the distance required for the forced wave over the shelf to become fully developed is large.

At a given location on the shelf a slowly developing, but potentially large, forced wave beneath a near-critical storm may or may not be larger than the rapidly developing but ultimately smaller forced wave beneath a storm that is far from critical. Figure 7b plots the displacement at the storm’s location relative to the inverted barometer response. For subcritical storms at \( 2L_s \) the forced wave is only 10%–30% larger than it was in deep water, with near-critical storms typically having slightly larger displacements. For supercritical storms the displacement under the storm decreases as the forced wave makes the transition from elevation in deep water to a depression in shallow water. Figure 7 shows that by \( L_s \) storms well above critical are significantly more developed, and it is not until \( \sim 4L_s \) that the magnitude of displacement under the near-critical \( F_r = 1.1 \) storm exceeds that of a storm with \( F_r = 1.6 \). Thus, the forced waves beneath subcritical storms that are near critical are typically slightly larger than storms with small \( F_r \) for distances up to \( 2L_s \). For supercritical storms those near critical have significantly smaller forced waves for distances up to \( 2L_s \) than those with large \( F_r \).

**d. Implications for coastal storm surge**

Although the focus of this work is the transient waves generated by storms, the model has implications for the development of coastal storm surge under fast-moving storms, in particular the enhancement of the forced wave or storm surge as it moves from deep water across a finite width shelf. Figure 8 shows the solution for a perfectly reflecting vertical coastal wall on a continental shelf \( 10L_s \) wide. Free waves reflecting off the coast have the same amplitude as the incident wave and, when the forced wave crosses the coast, it generates a transient, given by (6). Figure 8a shows the solution at
that arrives after $c$

Forced waves

$$A_d = \frac{1}{(1 - F_{r_d}^2) A_0}$$

$$A_c = \frac{1}{(1 - F_{r_c}^2) A_0}$$

Primary transients

$$A_{R1} = -\frac{1 - F_{r_d}}{(1 + F_{r_d}/F_{r_c}) A_0} \leftarrow c_d$$

$$A_{F1} = -\frac{1 + F_{r_d}}{(1 + F_{r_d}/F_{r_c}) A_0} \rightarrow c_s$$

$$A_{R2} = -\frac{1 + F_{r_c} - \Delta A_0}{(1 + F_{r_d}/F_{r_c}) A_0} \leftarrow c_s$$

$$A_{F2} = -\frac{1 + F_{r_c} - \Delta A_0}{(1 + F_{r_d}/F_{r_c}) A_0} \rightarrow c_d$$

Descendants of $\eta_{F1}$

$$A_{F1R1} = \beta A_{F1} \leftarrow c_d$$

$$A_{R1R2} = \alpha A_{R2} \rightarrow c_s$$

$$A_{F1R2} = \beta A_{F1R2} \leftarrow c_d$$

$$A_{R2R1} = \alpha A_{R1} \rightarrow c_s$$

Descendants of $\eta_{R2}$

$$A_{R2R1} = \beta A_{R2R1} \leftarrow c_d$$

$$A_{F2R1} = \beta A_{F2R1} \leftarrow c_d$$

The amplitude of the coastal storm surge is again dependent on the development of the forced wave under the storm after it crosses onto the shelf and thus is limited by shelf width. Figure 9 shows the maximum displacement at the coast for shelves up to $5L_s$ wide when $F_{r_d} = 0.2$. For subcritical storms the amplitude of the storm surge increases with $F_{r_c}$, with near-critical storms having the largest surges. The elevation under a supercritical storm is progressively reduced from its deep-water elevation as it moves across the shelf and becomes a depression if the shelf is wide enough. Thus, a supercritical storm might be expected to have a reduced storm surge. However, Fig. 9 shows large surges for supercritical storms. For supercritical storms on a wide shelf the surge is not due to the forced wave but due to the first forward transient $\eta_{F1}$ that arrives after the storm. From (11) $\eta_{F1}$ is positive as $\Delta < 0$ for supercritical storms. Thus for a transcritical storm $\eta_{F1}$ is a wave of elevation and produces a surge.

e. Temporal transients

Transients are also generated by a storm changing its intensity, translation speed, or direction: all of which change the forced pressure wave under the storm. Following the technique in Kundu and Cohen (2002), a 1D moving storm in uniform water depth instantaneously starting over a flat constant depth ocean will generate two temporal transients given by

$$\eta_T = -\frac{1}{2} (1 \pm F_r) \eta_S(x \mp ct),$$

where the steady-state response $\eta_S$ is given by (4). These transients ensure that the total surface displacement and its time derivative are both zero at $t = 0$. The numerical solution confirmed (19), which has a larger forward transient and a smaller reverse transient moving as free waves away from the storm’s initial location. Large temporal transients will be generated by a near-subcritical storm that accelerates slightly to a near-supercritical speed. Unlike topographic transients, temporal transients (19) have the same width as the storm and thus their characteristic time-scale $T_S' = L_s/c = F_r T_s$. Thus, for $F_r < 1$ temporal transients associated with instantaneous changes have shorter periods than topographic transients.
Fig. 7. Development of ocean surface displacement at the center of a Gaussian storm after crossing a step for $\text{Fr}_d = 0.2$. Labels indicate shelf Froude number $\text{Fr}_s$. Dark lines are subcritical storms and gray lines are supercritical storms. (a) Change in elevation under a storm after it crosses a step relative to the change in the amplitude of the steady-state forced pressure wave between deep and shallow water, and (b) elevation under storm.

Fig. 8. Ocean surface displacement for a Gaussian-shaped storm crossing a topographic step and coastline for $\text{Fr}_d = 0.2$ and $\text{Fr}_s = 0.7$ (thick black line is the analytic solution; overlying gray line is the numerical solution): (a) $t = 7T_S$, dashed–dotted line is the forced wave at $t = -3.5T_S$ and dashed line gives the forced wave and first forward transient at $t = 3.5T_S$; (b) $t = 10T_S$; (c) $t = 12T_S$; and (d) $t = 16.5T_S$. 
Changes in storm intensity and translation velocity are not instantaneous and are likely to take many hours. This would significantly reduce the amplitude of any temporal transients as well as give them much longer periods. Thus, though they clearly exist, temporal transients would generally be expected to be much smaller than topographic transients. Equation (19) does, however, demonstrate that forward temporal transients are larger than the reverse transients.

3. The 2D numerical model

The 1D analytical model of the previous section gives clear results but is limited as it does not allow for the 2D spread of the transient waves. Spreading causes transient amplitudes to decay inversely with distance as they move away from the storm. Also, supercritical storms in the 1D model cannot develop a barotropic wake as they do in the Mercer et al.’s (2002) 2D numerical model. To investigate 2D topographic transients a numerical model similar to that used by Mercer et al. was developed. Details of the 2D barotropic shallow-water model are given in the appendix. The model for small fast storms neglects Coriolis effects and is frictionless except in a “$2L_S$-wide sponge” layer around its edges, which was used to minimize wave energy reflected from the edges of the numerical domain.

a. Subcritical 2D storm crossing a ridge

The surface displacement along the storm track for a 2D storm crossing a topographic step with $Fr_d = 0.2$ and $Fr_s = 0.8$ (not shown) was similar to Fig. 2. Figure 10 gives displacement contours for a 2D forced wave crossing a ridge. The $0.2A_0$ depression of the first forward transient generated at the first step, $\eta_{F1}$, is seen moving ahead of the storm in Fig. 10a. The 1D model in Fig. 3 indicates for these Froude numbers that the initial size of $\eta_{F1}$ is approximately 50% the size of the shallow-water forced wave size. In 2D the transient spreads reducing its amplitude as it moves ahead of the storm in Fig. 10a. The 2D transients are not radially symmetric, radiating energy preferentially in the direction the storm is traveling.

Unlike the 1D steady-state response (4), the 2D steady-state response is not a scaled version of the inverted barometer response. For a circular storm the steady-state displacement contours are elliptical but are approximately circular for small Fr. This can be seen in the elliptical forced wave over the ridge in Fig. 10a returning to near circular as it crosses back into deep water in Fig. 10b. The forced wave moving over the second step in Fig. 10b creates two new transients, the small $\eta_{F2}$ moving out into deep water, while the $0.2A_0$ depression $\eta_{R2}$ spreads back over the ridge. The small reflection of $\eta_{F1}$, $\eta_{F1R1}$, is also seen spreading back across the ridge.

b. Supercritical 2D storm crossing a ridge

Figure 11 gives the displacement along the track of a supercritical storm that crosses a step. The steady-state
The forced wave now consists of a depression ahead of the storm’s center with elevated water levels behind the center, as noted by Mercer et al. (2002). The slow topographic transient generated at the step, \( \eta_{T1} \), is seen at the back of the storm in Fig. 11a and becomes distinct in Fig. 11b as it falls farther behind the storm. The reduction in amplitude of \( \eta_{T1} \) between Figs. 11a and 11b is due to its spread. From (18) with \( Fr_s = 1.4 \) at \( t = 10T_S \), the separation is \( \delta = 2.9L_S \), in agreement with Fig. 11b. As in the 1D model the transient is a smaller inverted copy of the steady-state forced wave. To enable the transient and forced wave to visually separate in the supercritical examples given in this section a large \( Fr_s = 1.4 \) was used: as a consequence the transient amplitudes are smaller than they would be for a near-critical storm.

Figure 12 gives surface elevation contours for a supercritical storm crossing a ridge. The “V”-shaped bow wave associated with the depression and elevation of the forced wave, discussed by Mercer et al. (2002), is clearly evident in Fig. 12a. The rear part of the 0.2\( A_0 \) elevation topographic transient, \( \eta_{T1} \), is distinct from the forced wave in Fig. 12b, consistent with partial separation, \( \delta = 1.4L_S \). As the storm crosses the second step back into deep water in Fig. 12b, a weak forward transient depression is radiated ahead of the storm and the forced wave returns to a near-circular shape. The forward transient is not just \( \eta_{T2} \) but also contains the transmitted component of the free wave depression at the front of the wake.

After leaving the ridge in Fig. 12c most of the free waves in the storm’s wake are reflected by the step as discussed in Mercer et al. Trailing behind the reflected wake at the same speed is the 0.2\( A_0 \) amplitude topographic transient \( \eta_{T2} \). A near-supercritical storm would give larger transients: the high \( Fr \) used in the example was chosen to allow the forced wave and transients to visually separate. For transcritical storms \( \Delta < 0 \), thus from Table 1 \( A_{R2} \) is positive, consistent with Fig. 12c which shows \( \eta_{R2} \) is a wave of elevation as indicated by gray shading. The small transient \( \eta_{T1} \) is not discernable in Fig. 12c from the larger reflected wake, but slightly after this time \( \eta_{T1} \) is partially reflected by the step and also radiates back across the ridge just behind \( \eta_{R2} \).

4. Discussion

a. Grand Banks

Mercer et al. (2002) give observations of two storms over the Grand Banks. Tropical Storm Helene crossed the middle of the Grand Banks traveling northeastward in September 2000. The second, Tropical Storm Jose, crossed the southwestern corner in 1999. Mercer et al. attributed the Newfoundland long-waves events to the reflection and/or refractive of the supercritical storms’ wake from the edges of the Grand Banks. The 500-km-long Grand Banks are only 40 m deep at their shallowest point and depth increases rapidly to 3000 m on the southern side of the Banks. Over much of the Grand Banks the shallow-water wave speed is less than 30 m s\(^{-1}\), making the two storms, which had average translation speeds of 30 m s\(^{-1}\), supercritical with \( Fr_s \approx 1.1 \). Mercer et al. estimated the storm size to be around 40 km, giving \( T_S \approx 30 \) min and \( A_0 \approx 0.3 \) m. Their model for a realistic bathymetry showed good agreement with the water levels observed at the Newfoundland coast as a result of the 30-min-period waves.

Helene’s track crossed 400 km or 10 storm widths of the supercritical Grand Banks, developing an extensive
wake over 5 h. From (18) with Fr = 1.1 and t = 10T_s, δ ≈ 0.9, η_F1 would be only partially separated from the storm forcing and would be difficult to distinguish from the forced wave and wake. The 0.1-m depression trailing Helene in Mercer et al.’s (2002) Figs. 6b and 6c may be part of η_F1. When Helene hit the northeast side of the Grand Banks its wake was reflected and these waves were refracted around the edge of the banks toward the Newfoundland coast. Mercer et al.’s numerical model results using real bathymetry contain both the barotropic wake, which they discuss, and the topographic transients discussed here. In the following paragraphs the results and figures for the idealized geometry models presented in this paper are used to discuss topographic transients on the Grand Banks. For example, Fig. 12 indicates that, as the storms left the banks, they would generate a reverse transient η_R2, which would form part of the wave energy that impacted the Newfoundland coast.

Tropical Storm Jose crossed the southeast corner of the Grand Banks in October 1999. Its track traversed only 150 km of the supercritical region of the Grand Banks, that is, four storm widths, over 1.5 h. Figure 7b indicates that, at four storm widths from the step, Jose had enough time for the elevation of its forced wave in deep water to become a depression over the supercritical Grand Banks. If one assumes the same Fr = 1.1 as Helene, Jose and η_F2 would be δ = 0.4L_s apart when leaving the Grand Banks. Thus any topographic transients would be difficult to distinguish from the reflected wake, so the long-wave energy at the Newfoundland coast during Jose may be partially due to topographic transients.

To allow the storm and transients to visually separate in the supercritical example in section 3b, a large Fr = 1.4 was used and consequently exhibited smaller transients. For Jose and Helene Fr ≈ 1.1 for which the 1D model in Fig. 3 indicates the initial amplitude of the transients should be similar to the steady-state forced wave on the Grand Banks. Two-dimensional spreading reduces the amplitude of the transients as they cross the banks, whereas the storm wake maintains its amplitude. As the storms leave the Grand Banks they generated a transient R_1 whose initial amplitude also is similar to the forced wave and wake on the banks. As the transient spreads, its amplitude decreases, as seen in Fig. 12c. The transient’s initial lateral extent is confined to the region near the storm, much less than the extensive wake. Thus most of the long-wave energy in the Helene long-wave event derives from the extensive wake with a smaller contribution from transient wave energy, mainly that generated when it moved off the banks. Jose did not have time to develop an extensive wake, thus topographic transients generated by the storm, both as it moved on and off the banks, may be more significant during this event. To fully understand the long waves generated by these two storms requires a 2D version of the analytic model presented here to determine the relative amplitude of the waves due to the storm wake and those due to topographic transients.

b. New Zealand

The seamounts of the Louisville Ridge northeast of NZ sit in 5000-m deep water and range in depth from 400 to 3000 m. The storms translating at 20 m s⁻¹ observed by Goring (2005) have δ = 0.3 m and Fr_L = 0.1 and for the shallowest seamount Fr_L = 0.3, giving Δ = 0.09. Thus any transients generated by this seamount will be small and will also decay in amplitude as they
spread from the seamount. In addition, for these 100-km-wide storms $T_s = 80$ min, which is longer than the observed 5–30-min wave periods. Temporal transients associated with instantaneous changes in storm intensity would have appropriate periods $T^T = 8$ min for $c = 200$ m s$^{-1}$. However, temporal changes are likely to occur over a much longer period, giving both smaller amplitude and longer period temporal transients. Even allowing for the directional nature of the transient wave and for some amplification of topographic or temporal transients as they decelerate over the NZ continental shelf, it appears unlikely that the observed 0.5-m NZ long waves are the direct result of storms moving over the ridge, changing intensity or translation velocity.

c. Shelf seiches

The models presented here show that storms crossing topography radiate transient waves; however, it is only in the exceptional circumstance of a near-critical storm, such as Helene and Jose, that these transients become large. Though generally small, the transients may be significant as a source of seiche energy. Seiches in coastal embayments and on shelves often have periods of tens of minutes to hours, the same range as topographic transients, and may persist long after the storm and its transients have past. The source of seiche energy may be local, that is, from a storm crossing onto the adjacent continental slope, or it could also be distant. Open-ocean storms crossing ridges and seamounts will radiate transients. These transients have periods that fall between those of long swell and tidal periods, a range in which there is typically little energy in the open ocean. Unless the ridge or seamount is near critical, the transients will contribute little energy to the open-ocean energy spectrum. However, when they impinge on a shelf or embayment they may excite seiche modes. Thus it is possible that in some areas topographic transients from distant storms crossing multiple ridges or seamounts are a source of seiche energy.

5. Conclusions

The simple model demonstrates that, as the forced pressure wave under a storm moves over changes in water depth, it generates long barotropic free waves. The amplitudes of the free wave transients are proportional to the change in the amplitude of the fully developed forced wave under the storm between deep and shallow water, $\Delta$. The transient waves radiate energy from the storm preferentially in the direction the storm is traveling and can be a harbinger for the approaching storm. The topographic transient waves have a time scale equal to the storm’s passage time, $L_s/U$, the same as the waves in the wake of a supercritical storm. However, topographic transient waves are generated by both sub- and supercritical storms.

A number of factors influence the amplitude of the transients. An important factor is the Froude number in shallow water, Fr. Large transients are generated by storms, which become near critical in shallow water. The largest transients are generated by only a small depth change in transcritical storms whose deep- and shallow-water Froude numbers are near unity. One surprising result is that a wide continental slope only slightly reduces the amplitude of the transient waves if there is a large change in water depth across the slope (Fig. 2b). This results from the inverse square relationship between forced wave amplitude and Froude number. Near-critical storms require a very wide shelf to fully develop the forced pressure wave (Fig. 7). Real shelves may only be one or two storm widths wide; thus, shelf or ridge width can limit the forced wave amplitude. The development of a coastal storm surge is also limited by shelf width, and for narrow shelves the forced wave and forward transient may be indistinguishable. However, near-critical storms generally develop larger surges than slow-moving storms (Fig. 9). For transcritical storms on a wide shelf the surge is not due to the forced wave but is caused by the first forward transient that arrives after the storm.

Changes in the storm’s translation speed or its direction also generate waves: however, for small fast storms these temporal transients are likely to be much smaller than topographic transients. Temporal and topographic transients generated by the Louisville Ridge can impact the NZ coast but are likely too small to be directly responsible for the NZ long-wave events. The observed 5–30-min periods are too short to be explained by topographic transients with a characteristic time scale of 80 min. If the transients have a range of periods, the longer period NZ waves may be seiches resulting indirectly from transients. Whether or not transients are responsible for the longer period NZ waves, topographic transients are a potential source of energy for seiches on other continental shelves. The wave patterns from the Mercer et al.’s (2002) numerical model for two fast storms crossing the Grand Banks contain wave energy from both barotropic storm wakes that they discuss and topographic transients discussed here. The 1D model and 2D numerical models in this paper indicate that the Newfoundland long-wave events are a combination of waves due to the supercritical storm’s barotropic wake and topographic transients of the transcritical storms generated as they moved on and off the
Grand Banks. For Fr = 1.1 the initial topographic transients of these two storms will be much larger than for the examples given from the 2D model, which have a higher Fr. The transient’s initial extent is confined to the area near the storm. Helene developed an extensive wake much larger than the storm, while Jose had less time to develop a wake. Thus, it is suggested that topographic transient wave energy was only a small part of the wave energy during the Helene event and may have been more significant during the Jose event. Mercer et al. did not look for topographic transients in their model results. To understand the relative importance of the wake and topographic transients a numerical model for subcritical storms crossing the Grand Banks will be developed in future work.

The 1D analytical model has the advantage of giving clear analytical solutions for the transient waves and their amplitudes. These amplitudes will be realistic near a 2D storm; however, radial spread causes transient amplitudes to decay with distance from the storm. Consequently, far from the storm, the 1D model can only indicate the relative amplitudes of the transients. Real topographies are more complex than that of the model and thus will not give such clearly delineated transients. However, the model is useful in that it indicates where and how the largest transients are generated. Future work will develop a 2D analytic model to understand the relative importance of wake and topographic transients waves and their relative rates of development and to investigate the influence of factors such as the angle of incidence of a storm relative to the topography, Coriolis, and wind stress have on transient amplitudes.

This paper demonstrates that moving storms generate long barotropic waves as they cross over topography. For small fast-moving storms the frequencies of these free waves fall between the frequency of long swells and tidal frequencies, a band typically believed to have little energy in the open ocean, except for rare large seismic tsunami events. Topographic transients are generated by more frequent storm events, but it is only in certain circumstances that the transient waves become large enough to be easily distinguished. Demonstrating their existence may lead to new observations identifying topographic transient events in a broader range of circumstances. Topographic transients are suggested as a source of energy for seiches on shelves and within embayments. The energy may come from a storm crossing the adjacent continental slope, and it is speculated that distant open-ocean storms may also be a source of seiche energy, which accumulates from many small transient waves generated by a single storm crossing multiple ridges or seamounts.

### APPENDIX

#### Numerical Model

To confirm the analytical results and obtain solutions, which include a continental slope between ocean and shelf, the variable water depth form of \( \text{(3) was solved numerically. In nondimensional form the 1D governing equation is} \)

\[
Fr_d \frac{\partial^2 \eta'}{\partial t^2} - \frac{\partial}{\partial x'} \left( h' \frac{\partial \eta'}{\partial x'} \right) = - \frac{\partial}{\partial y'} \left( h' \frac{\partial \eta'}{\partial y'} \right), \tag{A1}
\]

where \( x' = x/L_S, t' = t/T_S, h' = h/h_0, \eta' = \eta/\eta_0, \) and \( \eta_0 = \eta_0/\eta_0. \) This was solved using a basic five-point explicit scheme to step forward in time. The Sommerfield radiation condition was used to allow free transient waves to exit the numerical domain. The grid spacings were \( \Delta x' = 0.025 \) and \( \Delta t' = \Delta x' Fr_d \). The initial condition model was for the Gaussian storm’s steady-state deep-water response \( (8) \) at \( x' = -10, \) a condition which generated negligible start-up transients. The 2D numerical model was similar to the 1D model; that is,

\[
Fr_d^2 \frac{\partial^2 \eta'_{xy}}{\partial t^2} - \frac{\partial}{\partial x'} \left( h' \frac{\partial \eta'_{xy}}{\partial x'} \right) = - \frac{\partial}{\partial y'} \left( h' \frac{\partial \eta'_{xy}}{\partial y'} \right) = \frac{\partial}{\partial x'} \left( h' \frac{\partial \eta'_{xy}}{\partial x'} \right) = \frac{\partial}{\partial y'} \left( h' \frac{\partial \eta'_{xy}}{\partial y'} \right). \tag{A2}
\]

The explicit solution technique used a simple grid of seven points around a central grid point with \( \Delta x' = 0.025 \) and \( \Delta t' = \Delta x' Fr_d/2. \) The initial condition was a circular Gaussian displacement given by the steady-state deep-water response \( (8) \) at \( x' = -2. \) In 2D the steady-state response to a circular storm is not circular: however, for a deep-water start \( Fr_d = 0.2 \) was sufficiently small that the deep-water steady-state response is nearly circular. Only very weak start-up transients were generated by using a circular version of \( (8) \) as the initial condition. In 2D the Sommerfield radiation condition only ensures waves incident normal to the boundaries are not reflected. To minimize reflection for all angles of incidence a 2LS-wide linear frictional Rayleigh sponge layer was placed around the numerical domain (Durrant 1998).

### REFERENCES


and D. Walker, Eds., Adelaide, Australia, Institute of Engineers, Australia, 137–141.